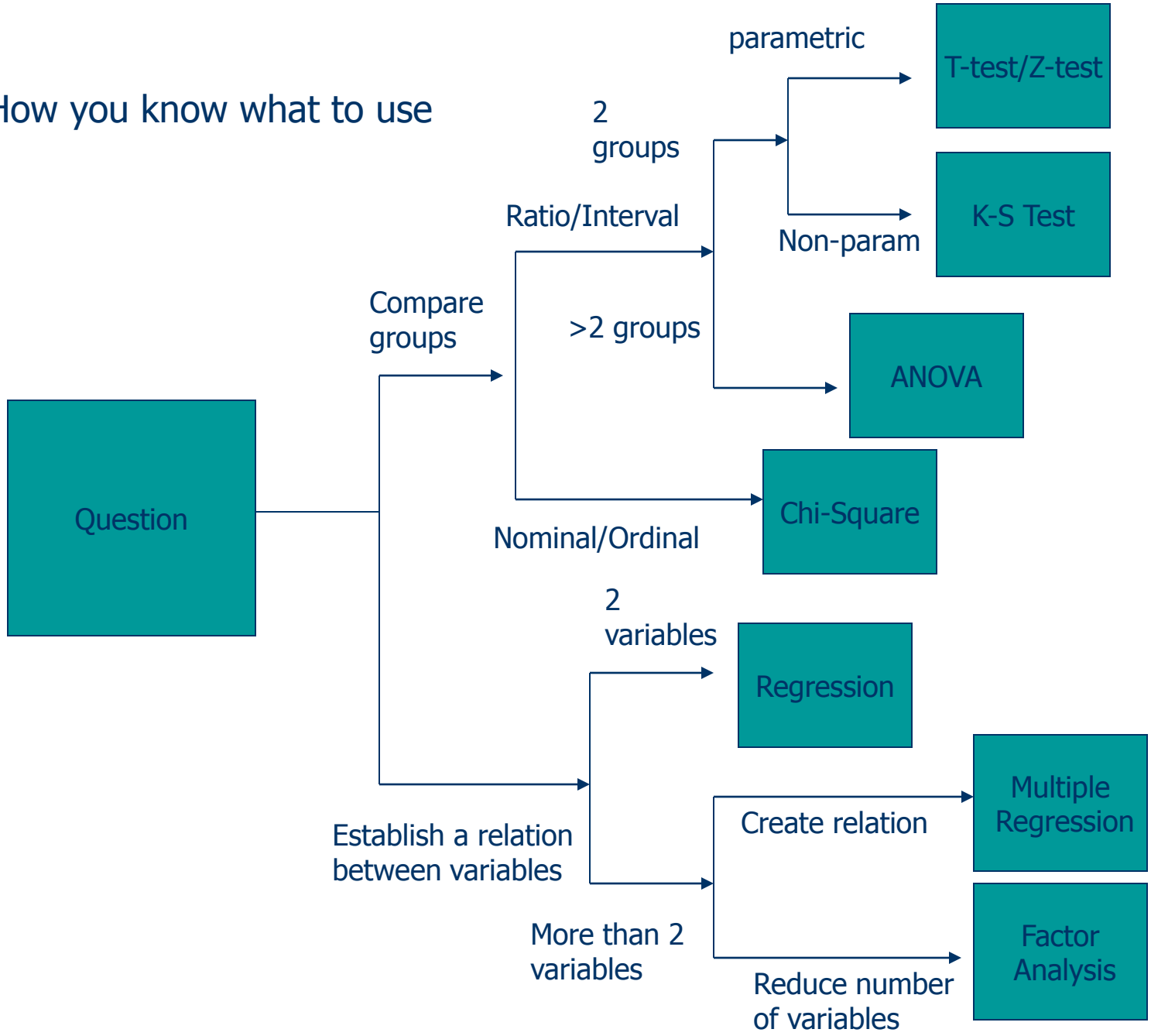
Inferential statistics defined

- Used to make an "inference" about the characteristics of the population
- Samples never perfectly characterize the population
- Therefore, there is error in our population inference

How you know what to use



Inferential tests

Inferential test	# of groups	Kind of data	What it does
t-test (independent)	2	Interval/ratio	Compares means (2 different samples)
t-test (dependent)	2	Interval/ratio	Compares means (2 related samples… often pre/post)
ANOVA	2+	Interval/ratio	Compares means (2+ different samples)
Regression	2+	Interval/ratio	Relates two or more variables (one dependent)
Factor Analysis	2+	Interval/ratio	Divides variables into groups that vary together
K-S Test	2	Interval/ratio	Compares two distributions to see if they are the "same"
Chi-Square	2+	Nominal/ordinal	Compares frequencies

Independent t-test

Is there difference in fracture density between two lithologies?

- We want to know the difference between two groups
- Key word='Difference'

Dependent t-test

Is there a difference between pre- and post-test sediment yields from a burned watershed?

- We have a control condition (before and after so we gain some statistical power)
- Key idea='Change' Have same system but but before and after

ANOVA

Is there a difference in the flattening of porphryoblasts (a to b axis ratios) in three parts of a shear zone?

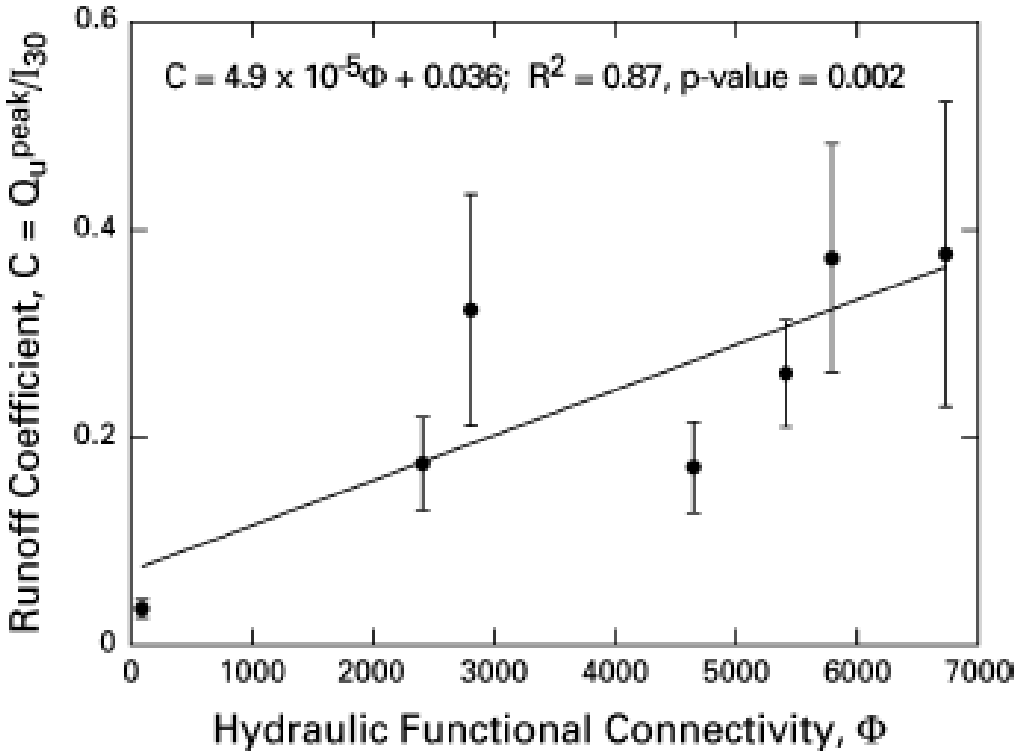
Key words: Difference and three (greater than 2)

Key point: You can only find that at least two of these 3 are different (but you don't know which two)

Correlation

Are burned severity and runoff related?

- Ask about relation as opposed to difference



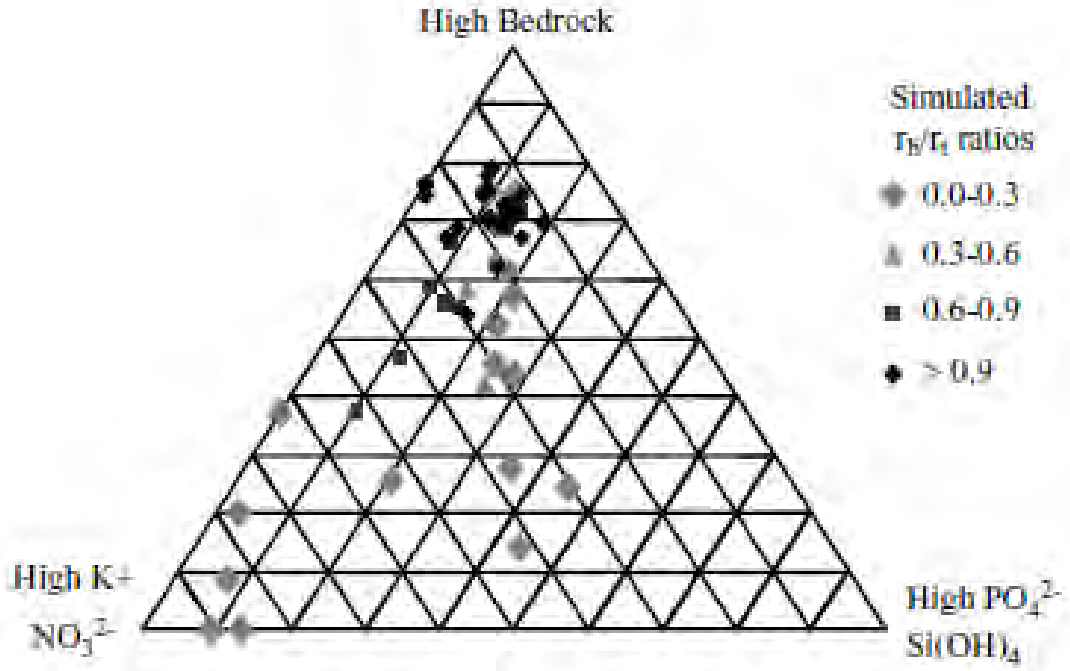
Multiple Regression

Can we predict erosion by knowing some combination of land use, soil and meterological variables?

- Write equation with so all of these variables in it that explains some variation of the dependent variable (erosion)
- Key words relation, multiple, predict (want an equation where we use some variables to give us others)

Factor Analysis

- What are the end-members in a chemical mixing problem? How do the data group?
  - Looks at data that vary in the same way and group them
  - If you have large datasets—this helps



T-test hypothesis test

- **Research hypothesis *(H_1)*:** Moisture content on north slopes is greater than moisture content on south slopes (Cannot be proven only supported) *One tail*

$$\mu_{north} > \mu_{south}$$

- **Research hypothesis that is not directional:** Moisture content on north slopes is not the same as moisture content on south slopes *Two tail*

$$\mu_{north} \neq \mu_{south}$$

- **Null hypothesis *(H_0)*:** There is no difference in moisture content between north and south slopes (try to reject this in order to support research hypothesis)

$$\mu_{north} = \mu_{south}$$

Hypothesis testing, cont'd

- Goal: We want to "reject" the null hypothesis
- Can never "prove" the research hypothesis
- Can't prove the null hypothesis—just need one instance to say it's wrong
- Can reject the null hypothesis

Next step… find the probability of being wrong in rejecting the null hypothesis

T-test hypothesis test

- Collect data on north and south slopes at the same times (days)
- Compare the means of the north samples to the south samples: assume the mean is the same (null hypothesis)

Is there a difference between group A and group B?

- Difference could be in means or standard deviation (variably defined)
- No proof of difference: we can prove that two things are not the same
- No absolute certainty

Hypothesis testing, cont'd

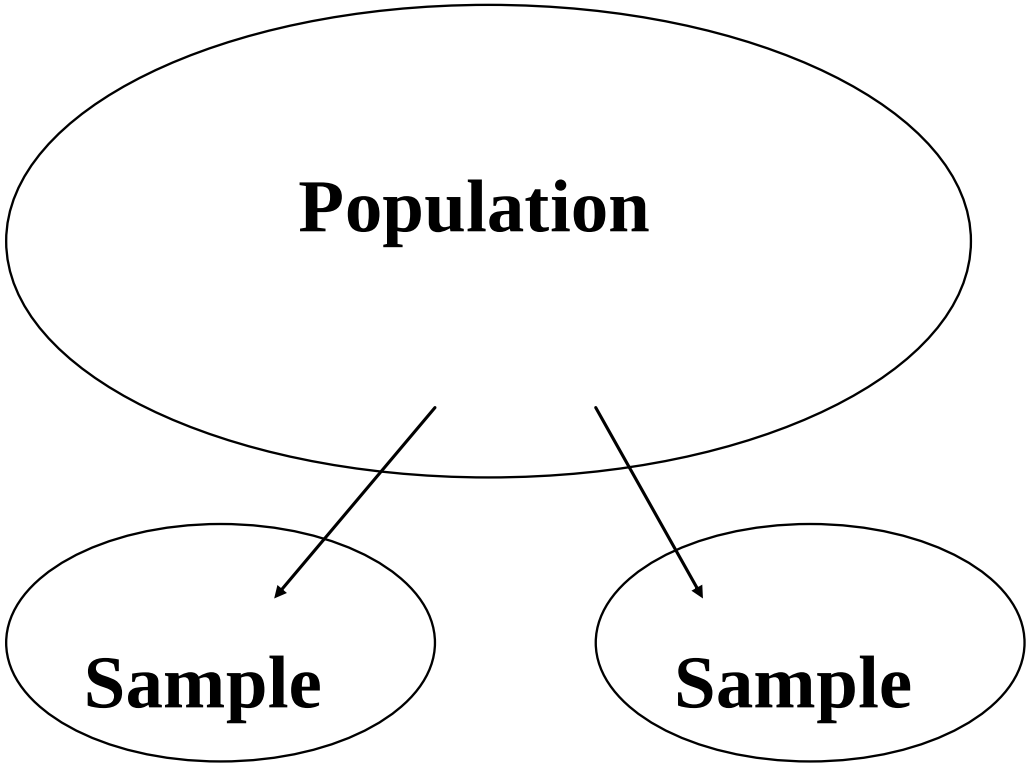
- α (alpha) or "*p*" to denote the probability of being wrong in rejecting the null hypothesis… we are testing "statistical significance"

- E.g., if *p*=.10, then we have a 10% probability that any differences between 2 groups is due to error, not due to true group differences.

Statistical significance

- amount of difference between group means (*p* is smaller when the differences between means is greater)
- *n* – easier to detect differences between means when our sample size is large
- how much error we have due to sampling or other factors (e.g., measurement error)

T-test



Are the two samples part of the same population?

T-test assumptions

- Normal distribution
- Approximately equal variances
- Usually less than 30 samples (use Z-test with similar assumptions if > 30)

Sample T-test

- Research hypothesis: is there a difference between mica ages in a valley and a mountain?
- Null hypothesis: the mountain and the valley are exactly the same

Data (Dates)

- Mountain
  - 279 MA
  - 292 MA
  - 291 MA
  - 288 MA
  - 285 MA
- Valley
  - 264 MA
  - 275 MA
  - 271 MA
  - 256 MA

Output from Excel

t-Test: Two-Sample Assuming Unequal Variances

	Mountain	*Valley*
Mean	287.00	266.50
Variance	27.50	69.67
Observations	5.00	4.00
Hypothesized Mean Diffe	0.00	
df	5.00	
t Stat	4.282	
P(T<=t) one-tail	0.004	
t Critical one-tail	2.02	
P(T<=t) two-tail	0.008	
t Critical two-tail	2.57	

Is liquid limit different between two soils?

Liquid limit—Williams Fork Lake

- 32.2
- 33
- 31.6
- 32.2
- 45.9
- 41
- 38.9
- 41.2
- 37.9
- 41.5
- 41.0
- 42.6
- 37.1
- 34.5
- 72.5

Liquid limit-Eldorado Lake

- 64.6
- 38.8
- 64.9
- 42.1
- 25.6
- 24.8
- 23.2
- 26.4
- 33.0
- 27.5
- 26.0
- 24.0
- 34.9
- 36.4
- 37.9

Excel Output

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	40.20667	35.34
Variance	99.17067	179.394
Observatio	15	15
Hypothesiz	0	
df	26	
t Stat	1.129313	
P(T<=t) on	0.134539	
t Critical on	1.705618	
P(T<=t) tw	0.269078	
t Critical tw	2.055529	

ANOVA: Single Factor

- Analysis of Variance: Different Types
- Uses F-distribution (rather than T)
- Approximately equal variances
- Compares variation within a group to between groups (ratio between the two)
- Again critical to look at multiple sets of data
- Can use F-test to compare variances

Comparison

Infiltration Rates: My Experiments

	Test 1	Test 2	Test 3	Test 4
S13	28.3	36.08	27.67	18.39
S15	20.13	17.85	26.49	18.65
N13	18.24	18.54	21.31	
N15	20.65	23.73	22.5	

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Row 1	4	110.44	27.61	52.40967
Row 2	4	83.12	20.78	15.3828
Row 3	3	58.09	19.36333	2.864633
Row 4	3	66.88	22.29333	2.403633

ANOVA

rce of Varia	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between G	145.0361	3	48.34537	2.260038	0.1439	3.708265
Within Gro	213.9139	10	21.39139			
Total	358.95	13				

Summary Hypothesis Testing

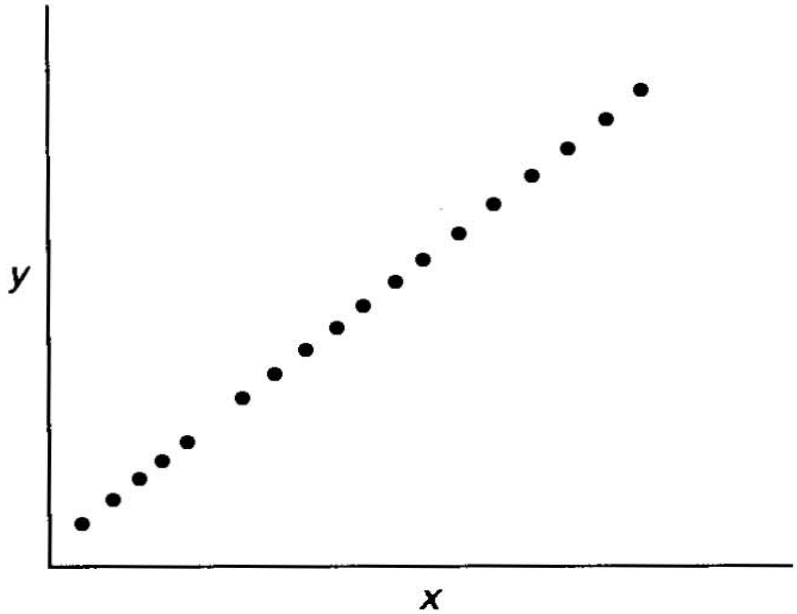
- Know which test(s) to use in which situation and more importantly whether you are trying to show a difference or a relation
- Your method depends on your question!
- Apply the tests but be aware of assumptions
  - Normality
  - Equal Variances
- Always plot data and compare descriptive statistics before moving forward. Remember something might not be statistically significant but still may be important!

Correlation

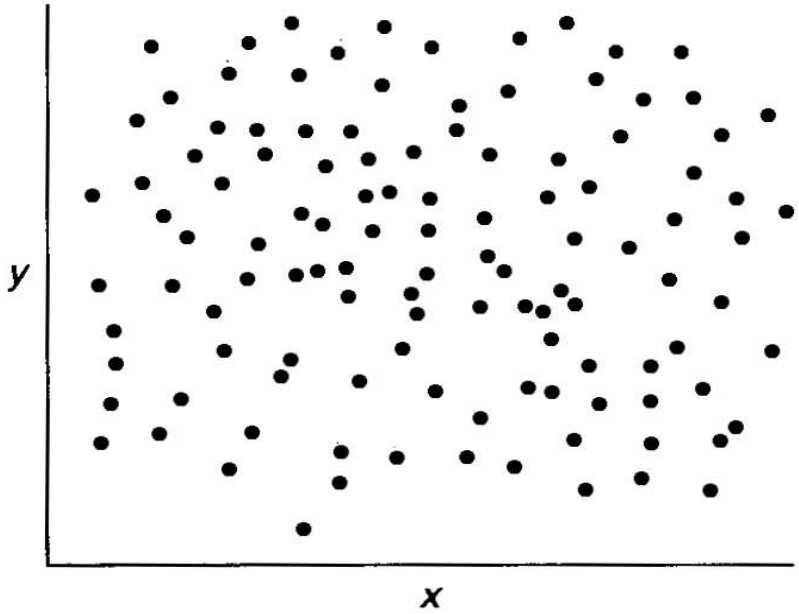
- How related are 2 or more variables?
- More specifically, correlation is used for…
  - Knowing the extent to which one variable is related to another variable. If a participant has a high score on one variable is that person likely to have a high score on the other variable?
    - Do the variables "vary" together?
  - Knowing the direction of the relationship between variables
    - How do the variables "vary" together?

Correlation

- Value of one variable predicts the value of another variable. Correlation indicates both the strength of the association and its direction
- Scale from -1 to +1
- No units



Perfect Positive Correlation
($r = +1.00$)



No Correlation
($r = .00$)

Coefficient of determination (r^2)

- r^2 represents the % of common variation between 2 variables
- If r=.50, then r^2=.50*.50=.25
  - This means that 25% of the differences in one variable can be "explained" by the other variable.

Correlation does NOT imply causation

- One variable can be strongly related to another variable yet not cause it

- For causation, we need:
  - Strong correlation
  - Presumed cause comes before presumed effect
  - Elimination of outside variables

Distorters…

- Restricted range
- Outliers
- Non-linear relationships

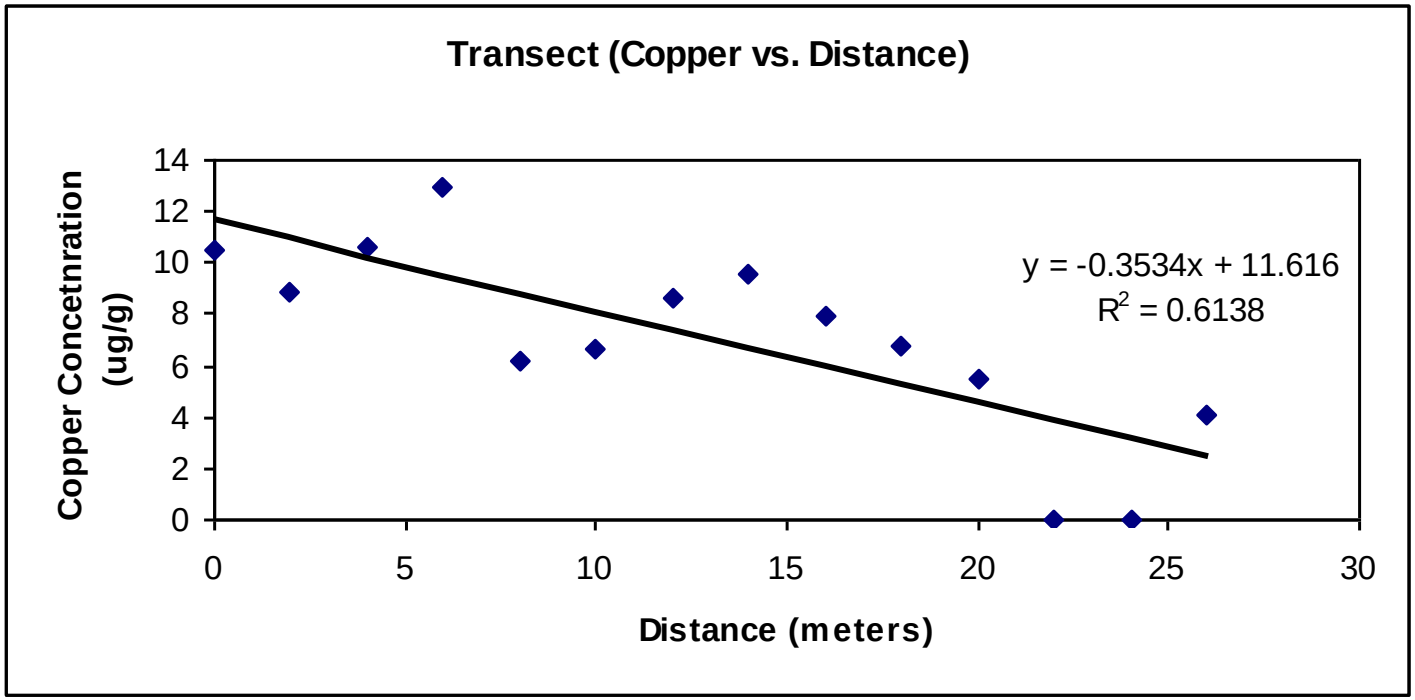
When interpreting *r*, think about:

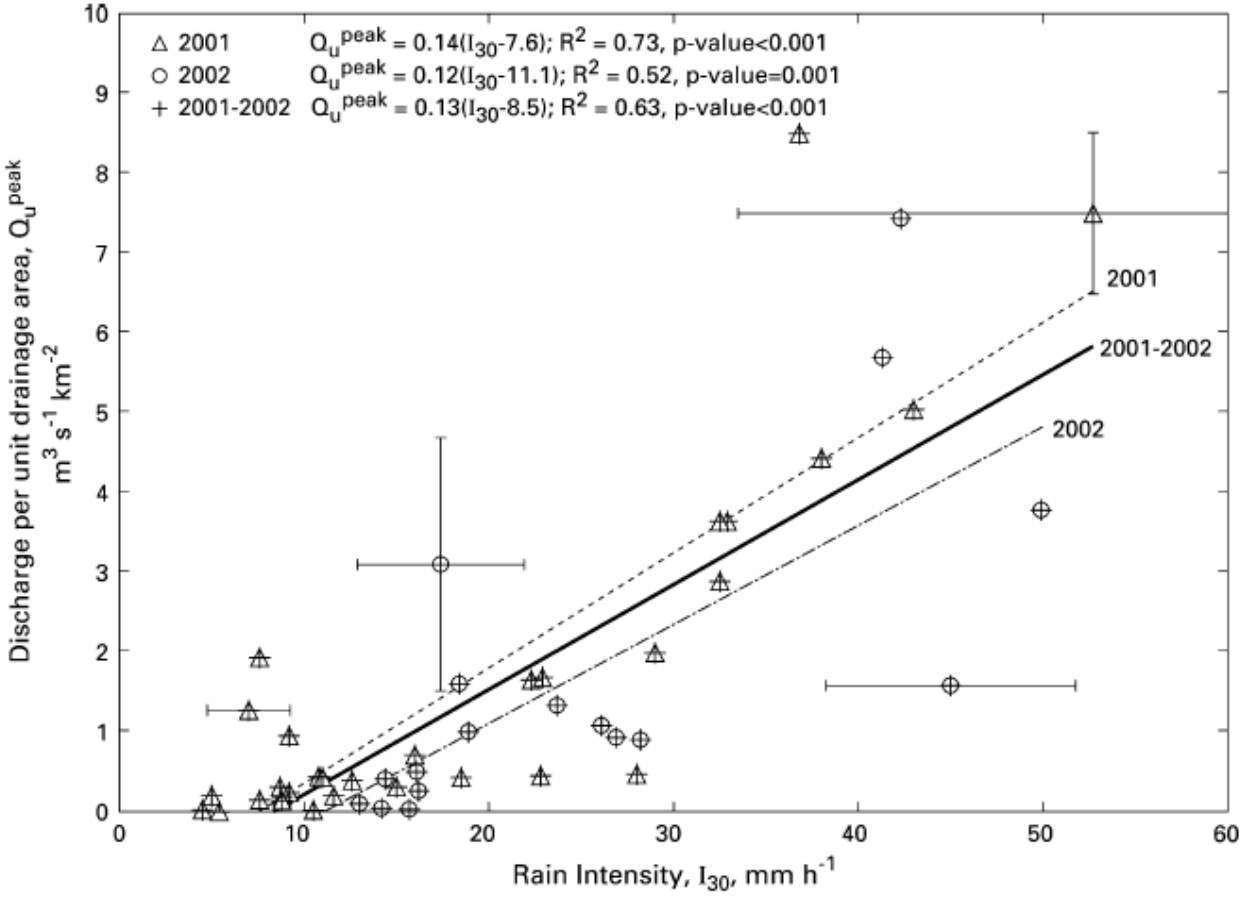
- Sign: direction of relationship

(+ or -)

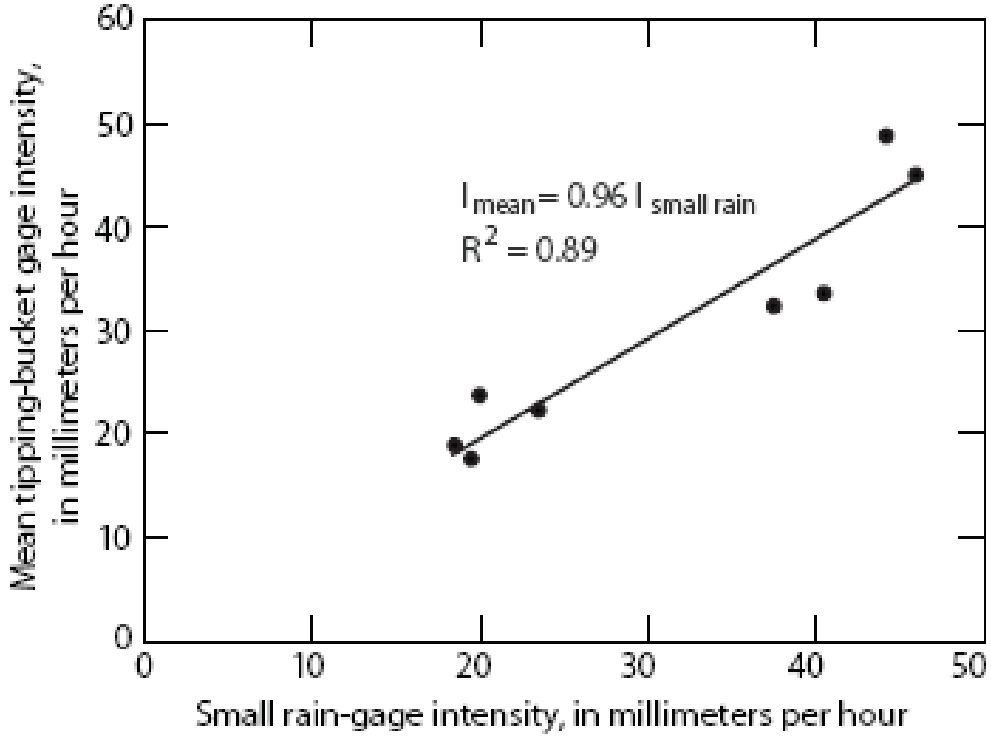
- Magnitude: strength of relationship
  - r=.1-.3 (weak)
  - r=.4-.6 (moderate)
  - r=.7+ (strong)
- Sample size

	Distance	Ca	Cu	Fe	Mg	Mn	Zn	K	Sand	Silt	Clay
Distance	1.00	-0.16	-0.78	-0.41	-0.37	0.49	-0.49	0.58	-0.68	0.42	0.46
Ca		1.00	0.34	0.21	0.91	0.10	0.52	0.51	-0.42	0.67	0.62
Cu			1.00	0.27	0.51	-0.49	0.35	-0.36	0.41	-0.07	-0.12
Fe				1.00	0.43	0.49	0.81	0.25	0.29	-0.24	-0.14
Mg					1.00	0.00	0.65	0.38	-0.21	0.46	0.38
Mn						1.00	0.38	0.73	-0.43	0.27	0.39
Zn							1.00	0.22	0.08	0.09	0.10
K								1.00	-0.71	0.62	0.70
Sand									1.00	-0.90	-0.86
Silt										1.00	0.93




△ 2001 Q_u^{peak} = 0.14(I_{30}-7.6); R^2 = 0.73, p-value<0.001
○ 2002 Q_u^{peak} = 0.12(I_{30}-11.1); R^2 = 0.52, p-value=0.001
+ 2001-2002 Q_u^{peak} = 0.13(I_{30}-8.5); R^2 = 0.63, p-value<0.001
2001
2001-2002
2002
Discharge per unit drainage area, Q_u^{peak} $m^3 s^{-1} km^{-2}$
Rain Intensity, I_{30}, mm h^{-1}
0
1
2
3
4
5
6
7
8
9
10
0
10
20
30
40
50
60

Example


Mean tipping-bucket gage intensity, in millimeters per hour
Small rain-gage intensity, in millimeters per hour
$I_{mean} = 0.96\, I_{small\ rain}$
$R^2 = 0.89$
0
10
20
30
40
50
60